Stochastic finite element analysis of 3D thin-walled structures

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Abstract

In this paper new method of analysing nonlinear systems, consisting of thin-walled bars with imperfections is presented. Description of thin-walled bars is based on modified Gruttmann-et al element [1] . In stochastic analysis Monte Carlo simulation and stochastic finite element method are applied, which are presented by Shinozouka et al [3,4]. This method can be used in engineering practice to analyse geometrical and material nonlinear steel structures with imperfections.

Keywords: computational mechanics, stochastic finite element method, thin-walled structures, Monte Carlo simulation,

1. Introduction

The theory and methods of stochastic analysis have been developed significantly during the last ten years. Most of the studies were made to develop methods of quick integration of probability integrals by means of first and second-order reliability methods, to extend knowledge about perturbation methods, sensitivity analysis and optimisation in order to find the design point of reliability. In this paper dissimilar approach is presented. It is tried to show that modified Monte Carlo method of numerical simulation is equally efficient, especially when applied to bars structures, and because of its conceptual simplicity it can be easily implemented in engineering practice without necessity of introducing concepts of second-order reliability methods. Designing of steel structures involves mainly analysis of structures consisting of thin-walled bars with imperfections and this is the subject of presented paper.

2. Formulation for thin-walled element

Finite element formulation for arbitrarily curved thin-walled bars is presented in accordance with Gruttmann approach [1], in which Timoshenko element was modified, taking into account effect of rotation and buckling of sections across bar's centre of gravity independent of linear displacements.

In presented paper scope of investigations is limited to small rotations caused by bending and moderately small rotations caused by torsion, which enables to obtain explicit form of deformation tensor. As a result, in limiting cases, e.g. for straight or circular bars, the solution is consistent with classical approach of engineering mechanics. To take into consideration finite rotations, co-rotational element should be analysed, which takes into account the fact, that large rotations are caused by rigid movement of the element [2]. Hence, it can be stated that presented approach can be also used in analysis of large rotations.

The position vectors of point M: $\mathbf{R}_M(S,\xi_2,\xi_3)$ of undeformed and $\mathbf{r}_M(S,\xi_2,\xi_3)$ of deformed cross-section in the space (S,ξ_2,ξ_3) (S=[0,L] is arc-length parameter of the spatial curve) are given with the following kinematic assumptions:

$$\mathbf{R}_M(S,\xi_2,\xi_3)=\mathbf{R}(S)+\xi_2\mathbf{E}_2+\xi_3\mathbf{E}_3\,, \tag{1a}$$

$$\mathbf{r}_M(S,\xi_2,\xi_3)=\mathbf{r}(S)+\xi_2\mathbf{t}_2+\xi_3\mathbf{t}_3+\omega(\xi_2,\xi_3)\alpha(S)\mathbf{t}_1(S) \tag{1b}$$

where t_i is assumed to be a piecewise constant. The given sector coordinates $\omega(\xi_2,\xi_3)$ and amplitude of warping $\alpha(S)$ is defined within Vlasov constrained torsion theory for bars. The cross-sections of the beam therefore lie in planes described by the basis vectors $[\mathbf{E}_2,\mathbf{E}_3]$.

Transformation of vectors $\mathbf{t}_i(S)$ after deformation and $\mathbf{E}_i(S)$ before deformation occur in accordance with following equations (2):

$$\mathbf{t}_1=(1-\theta_2^2/2-\theta_3^2/2)\,\mathbf{E}_1+\theta_3\,\mathbf{E}_2-\theta_2\,\mathbf{E}_3\,, \tag{2a}$$

$$\mathbf{t}_2=(-\theta_3\cos\varphi+\theta_2\sin\varphi)\mathbf{E}_1+(\cos\varphi-\theta_3^2/2)\mathbf{E}_2+(\sin\varphi+\theta_2\theta_3/2)\mathbf{E}_3\,, \tag{2b}$$

$$\mathbf{t}_3=(\theta_3\sin\varphi+\theta_2\cos\varphi)\mathbf{E}_1+(-\sin\varphi+\theta_2\theta_3/2)\mathbf{E}_2+(\cos\varphi-\theta_2^2/2)\mathbf{E}_3\,, \tag{2c}$$

where $\theta_2(S)$ and $\theta_3(S)$ are the bending rotations and $\varphi(S)$ is torsion angle.
The components of Green-Lagrangian strain tensor can be written as:

$$\mathbf{E}=\begin{bmatrix}E_{11}\\2E_{12}\\2E_{13}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}(g_{11}-G_{11})\\g_{12}-G_{12}\\g_{13}-G_{13}\end{bmatrix},\ g_{ij}=\mathbf{g}_i\cdot\mathbf{g}_j\,,\ G_{ij}=\mathbf{G}_i\cdot\mathbf{G}_j \tag{3a,b,c}$$

with the metric coefficients:

$$\mathbf{G}_1=\mathbf{R}^{(1)}+\xi_2\mathbf{E}_2^{(1)}+\xi_3\mathbf{E}_3^{(1)}\,, \tag{4a}$$

$$\mathbf{G}_2=\mathbf{E}_2\,,\quad \mathbf{G}_3=\mathbf{E}_3\,, \tag{4b,c}$$

$$\mathbf{g}_1=\mathbf{r}^{(1)}+\xi_2\mathbf{t}_2^{(1)}+\xi_3\mathbf{t}_3^{(1)}+\omega(\xi_2,\xi_3)\alpha^{(1)}\mathbf{t}_1\,, \tag{4d}$$

$$\mathbf{g}_2=\mathbf{t}_2+\omega_{,2}\alpha\mathbf{t}_1\,,\ \mathbf{g}_3=\mathbf{t}_3+\omega_{,3}\alpha\mathbf{t}_1\,, \tag{4e,f}$$

of the reference configuration and current configuration respectively. Index $^{(1)}$ denotes the customary symbol for differentiations with respect to the arc-length S. Symbols $\omega_{,2}$ and $\omega_{,3}$ stand for partial derivatives of warping function with respect to ξ_2 and ξ_3 coordinates, respectively.

3. Stochastic finite element of thin-walled bar

Formulation of stochastic finite element method is made in accordance with Shinozouka approach presented in [3,4], by means of Monte Carlo Simulation (MCS) and Neumann expansion for stochastic finite element equations of 3D thin-

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walled structures. Requirement of the Neumann expansion method that the absolute values of all eigenvalues of matrix **X** is given as

$$\mathbf{X} = \mathbf{K}_0^{-1}\Delta\mathbf{K} \tag{5}$$

for solution of stochastic finite element problem

$$(\mathbf{K}_0 + \Delta\mathbf{K})\mathbf{u} = \mathbf{P}\ , \tag{6}$$

should be less than 1, can always be satisfied, even for large components of the deviation matrix compared to the corresponding components of $\mathbf{K}_0$, after a modification proposed in [5].

Universally used perturbation methods, in which expansion stochastic system matrix to the Taylor series expansion and first-order approximation (FORM) or second-order approximation (SORM) [6,7] are used, are replaced by spectral representation method, in which stochastic field expands as a sum of trigonometric functions with random phase angles and deterministic amplitudes, presented in [8] in SFEM analysis of elastoplastics space frames made of Euler-Bernoulli bars.

4. Applications

The effectiveness of proposed method is tested through several numerical examples, using abovementioned FEs of thin-walled structures. Both the probability density function (PDF) and the first two statistical moments are obtained for a response surface. The results are compared with those provided by the classical MCS and Perturbation Method (FORM and SORM). Not only the accuracy of results is compared, but mainly effectiveness of the methods, measured by number of iterations necessary to obtain acceptable result in terms of engineering practice and as well by CPU Time. Calculations are carried out for the example presented in [9], concerning tapered beam with uncertain Young's modulus, approximated by finite plate's and shell elements. In presented paper model of thin-walled bar, which has warping degree of freedom, is used.

When performing a comparison analysis, in the perturbation approach, a sensitivity method of analysing thin-walled bars is implemented, described in [10].

What is more, increasing of random dispersion of construction's response and its load capacity with increasing number of stochastic finite elements is analysed, which effect is proved by Chodor (1997) in paper [11].

Results of aforementioned example will be presented in complete text of the paper.

5. Conclusions

This paper presents a methodology for accurate and efficient estimating of a response surface and reliability of stochastic finite element systems with application to space, thin-walled structures.

In general opinion Monte Carlo method is computationally expensive and this is the reason, why it is not used in important practical engineering applications. In this paper it was shown that against aforementioned opinion after using the spectral method, which decreases the number of necessary simulations and possibly Newman series expansion, which increases the computational speed, we obtain SFEM method, which is highly efficient and competitive with the perturbation method, which can be described by complex concept and difficult calculations of the system's sensitivity. Simple extension of the deterministic algorithms and in principle no necessity of implementation of optimising procedures to find the reliability index Hasofera-Linda β, indicates additional advantages of the method.

In case of systems consisting of bar's elements of relatively small number of degrees of freedom, presented method is very efficient and conceptually accessible for broad range of engineers. Also due to high and sufficient power of nowadays personal computers, presented method can be successfully used in everyday engineering practice.

Because steel structures are mostly built of thin-walled bars, presented method can be called as engineering method of estimating design displacement, cross-section's forces, limit points, load capacity and reliability of steel structures with geometrical, material and structural imperfections. Reliability index defined by standard [12] can be calculated based on estimated failure probability p_f according to dependence $\beta = -\Phi^{-1}(p_f)$, where Φ^{-1} is the inverse standardized normal distribution.

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